Interference Effects in Relativistic Inclusive Deuteron Electrodisintegration*

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Abstract

We extend the relativistic plane–wave impulse approximation formalism to incorporate a specific class of relativistic interference effects for use in describing inclusive electrodisintegration of ^{2}H. The role of these "exchange" terms for the various response functions accessible in parity–conserving and –violating inclusive processes is investigated and shown, especially for the latter, to have important consequences for experiment. An extension to a simple quasi–deuteron model is also considered.

1. Introduction

One of the long–standing theoretical problems in the study of electron scattering processes is the incorporation of relativistic effects in the modeling [1, 2, 3, 4], in part due to our incomplete knowledge of a relativistic Hamiltonian that realistically describes the strong interaction dynamics [5, 6] and in part due to the technical difficulty of solving such problems involving the relativistic many–body problem. On the other hand, the electromagnetic part of the interaction can — at least in principle — be described in a covariant way. It is the requirement that the level of the nonrelativistic approximations used in the description of the strong interaction aspects of the reaction and the one used in the electromagnetic operators be consistent that calls for the truncation of the nonrelativistic expansion of the one– and two–body electromagnetic operators to be used between nonrelativistic nuclear states [7, 8, 9]. For low– and even medium–energy studies of nuclei this has largely proven to be a successful way to proceed. However, there is increasing capability for electron scattering experiments at high energies and momentum transfers where relying on leading–order nonrelativistic approximations seems rather dangerous [10, 11, 12].

While awaiting the advent of realistic relativistic nuclear wave functions one might hope that kinematical regions exist where the relativistic effects that occur when probing a nucleus with a high momentum transfer virtual γ (or Z^0, see below) could largely "decouple" from the strong interaction part of the interaction, namely, from the nuclear transition matrix elements. The quasielastic (QE) region seems to serve that purpose: for a given momentum transfer q the energy transfer ω is so chosen as to transfer the 4–momentum essentially to one nucleon at the time, that is, along the kinematic line where $\omega = |Q^2|/2M$ with $Q^2 = \omega^2 - q^2$ and $M =$ nucleon mass. As a result, the roles of meson–exchange currents (MEC) and final–state interactions (FSI) are minimized [8, 9, 12]. Upon making two extra assumptions, namely that of a plane–wave final state and of incoherently summing over all single–nucleon contributions, it can be shown that the cross section factorizes into a product of a single–nucleon half–off–shell cross section and a spectral function, the latter basically describing the probability of finding a nucleon with a given energy and momentum in the target nucleus [13, 14]. This is the factorized plane–wave impulse approximation (PWIA). The relativistic corrections enter this model in a very different way for the spectral function and the single–nucleon cross section. In particular, whereas the scale that governs the relativistic effects in the former is basically dictated by the ratio of the Fermi momentum p_F to the nucleon mass, namely $\eta_F \equiv p_F/M$ which ranges from about 0.06 in the deuteron to 0.28 in heavy nuclei, the latter directly contains the effects of the high momentum transfer and so involves the ratio $\kappa \equiv q/2M$ which can be large — indeed "high momentum transfer" may be defined as $\kappa \sim$ unity or larger. To the extent that this model is roughly correct, the important relativistic effects associated with high momentum transfer can be addressed by treating the single–nucleon part relativistically while relying on traditional nonrelativistic descriptions of the spectral function. In a series of recent papers [9, 15, 16] we have pursued these ideas. In particular, the validity of the PWIA was tested by comparing it with a nonrelativistic calculation. When supplemented by earlier studies [8] that verify the suppression of the role of MEC in the QE region, these comparisons have led us to conclude that the model is suitable for describing high momentum transfer quasifree processes for parity–conserving electrodisintegration, if not for parity–violating electrodisintegration under all circumstances (see Ref. [9] and below).

On the other hand, when these ideas are carried to non–quasifree kinematics, in comparisons between the PWIA and more complete calculations the observed discrepancies are often interpreted in terms of the lack of FSI and MEC effects in the former. However, it is also relevant to ask how much of this discrepancy can in fact be attributed to the other assumption of the PWIA model, namely that of incoherence. One argument given is that for quasifree kinematics the interference effects should be suppressed by an extra power of Q^2 [14, 17]. Whereas this renders the contribution of such interference effects insignificant for deep–inelastic scattering, it still leaves the question open for scattering in the vicinity of the QE peak at high but not asymptotic momentum transfers, where it may still be important and in fact compete with the relativistic corrections we may wish to incorporate by means of the PWIA. Another argument invoked is that the interference effects in coincidence processes involve the amplitude where the virtual photon interacts with the $A-1$ residual nuclear state and would therefore be suppressed due to the rapidly falling form factor of this state [14] if this daughter system "sticks together". While this may be true for heavy nuclei at low missing energy, it obviously does not apply to few–body systems and it likely does not apply as well to heavy nuclei at high missing energy.

The purpose of this paper is to investigate the significance of these "exchange" effects for the deuteron in a simple, but relativistic model which is an extension of the PWIA in a way that includes such interference terms. Although our goal is to apply this formalism to inclusive scattering, we first obtain the coincidence response functions and then integrate over the detected nucleon's quantum numbers to obtain the inclusive answer. In so doing, we will recover the plane–wave "Born" approximation (PWBA) of Fabian and Arenhövel [8, 18], although our answer will be an extension of that earlier work as we include terms in all orders in $1/M$ for the single–nucleon electromagnetic current. We then calculate the inclusive responses and provide a simple interpretation for the significance of exchange effects in inclusive electrodisintegration of deuterium. We especially wish to clarify the roles of the kinematics and the form factors in suppressing or enhancing the importance of the exchange terms and to stress the important part that these terms play in forward–angle, parity–violating electron scattering. It is in fact this observation, combined with the attention drawn recently to high momentum transfer PV electron scattering experiments from few–body systems aimed at measurements of the strangeness form factors [19], that provides one of the main motivations for this work.

Finally, we consider a simple extension of our approach to include a treatment of a relativistic version of the quasi–deuteron model. Our interest in the present work where this model is concerned is rather focused: we explore the nature of the interference terms and relativistic effects in a situation where η_F, being connected as it is to the characteristic Fermi momentum for a many–body nucleus, is much larger than it is for the deuteron.

We have organized this paper as follows: in Sec. 2, we review the basic formalism for electron scattering in the PWIA and then in Sec. 3 we modify one of the assumptions that lead to the PWIA and obtain a variation of it which, following Arenhövel [18], we call "the relativistic plane–wave Born approximation" (relativistic PWBA). In Sec. 4 we investigate the role of the interference contributions for different response functions in inclusive parity–conserving and – violating electron scattering; additionally, the nonrelativistic limit of the PWBA is obtained in order for the physics to become transparent. In Sec. 5 the interplay between the interference and relativistic effects is explored for a wide range of kinematics and in Sec. 6 we briefly apply our formalism to the quasi–deuteron model. Finally, in Sec. 7 we present our summary and conclusions.

2. The Plane–Wave Impulse Approximation (PWIA)

Let us begin with a brief review of the coincidence electron scattering formalism within the context of the PWIA for a general nucleus; later we specialize to the case of deuterium. More detailed derivations and discussions can be found in the literature [15, 20, 21, 22]. The kinematics for electrodisintegration processes are presented in Fig. 1. Here an electron with four–momentum $K^\mu \equiv (\epsilon, \boldsymbol{k})$ is scattered through an angle θ_e to four–momentum $K'^\mu \equiv (\epsilon', \boldsymbol{k}')$. We restrict ourselves to one–photon exchange and plane–wave electrons. For the hadronic variables we have: $P_i^\mu \equiv (E_i, \boldsymbol{0})$ and $P_B^\mu \equiv (E_B, \boldsymbol{p}_B)$. Both energies are "on–shell", that is $E_i = M_i$ and $E_B = \sqrt{\boldsymbol{p}_B^2 + M_B^2}$; moreover, the outgoing on–shell nucleon has $P_N^\mu = (E_N, \boldsymbol{p}_N)$ with $E_N = \sqrt{\boldsymbol{p}_N^2 + M^2}$. The four–momentum transfer is $Q^\mu = (\omega, \boldsymbol{q})$ where $\omega = \epsilon - \epsilon'$ and $\boldsymbol{q} = \boldsymbol{k} - \boldsymbol{k}'$. We will denote the magnitude of the momentum transfer by $q = |\boldsymbol{q}|$.

The cross section for exclusive electron scattering in the laboratory system can be written as [20]

$$\frac{d\sigma}{d\Omega_e d\epsilon' d\Omega_N} = \frac{2\alpha^2}{Q^4} \left(\frac{\epsilon'}{\epsilon} \right) \frac{p_N M M_B}{(2\pi)^3 M_i} f_{rec}^{-1} \eta_{\mu\nu} W^{\mu\nu}. \tag{1}$$

Here $f_{rec} = 1 + \frac{2\epsilon}{M_i} \sin^2 \frac{\theta_e}{2}$ is the hadronic recoil factor and $\eta_{\mu\nu}$ is the leptonic tensor

$$\eta_{\mu\nu} = K_\mu K'_\nu + K'_\mu K_\nu + \frac{1}{2} Q^2 g_{\mu\nu} - i h \epsilon_{\mu\nu\alpha\beta} K^\alpha K'^\beta, \tag{2}$$

with $h = \pm 1$ the electron helicity (only longitudinally polarized electrons enter in the relativistic limit $m_e/\epsilon \to 0$; see Ref. [21]). The hadronic tensor $W^{\mu\nu}$ contains all information about the nuclear structure and dynamics and is constructed from the electromagnetic (EM) current transition matrix elements as

$$W^{\mu\nu} = (J_{fi}^\mu)^* (J_{fi}^\nu) = \overline{\sum}_{i,f} \langle f|\hat{J}^\mu|i\rangle^* \langle f|\hat{J}^\nu|i\rangle , \tag{3}$$

where $|i\rangle$ represents the target state and $|f\rangle$ the final state containing the residual nucleus and the emitted nucleon. The contraction of the two tensors can be written as

$$\eta_{\mu\nu} W^{\mu\nu} = \frac{1}{2} v_0 [R_{fi} + h R'_{fi}] , \tag{4}$$

where $v_0 = 4\epsilon\epsilon' \cos^2 \frac{\theta_e}{2}$. The quantities R_{fi} (R'_{fi}) contain the parts of the tensors in Eqs. (2) and (3) that are, respectively, symmetric and antisymmetric under the interchange of μ and ν. It can be shown that they can be decomposed in a sum of response functions multiplied by leptonic kinematical factors [21]:

$$R_{fi} = v_L R_{fi}^L + v_T R_{fi}^T + v_{TL} R_{fi}^{TL} + v_{TT} R_{fi}^{TT} \tag{5}$$

$$R'_{fi} = v_{T'} R_{fi}^{T'} + v_{TL'} R_{fi}^{TL'} . \tag{6}$$

Different responses contribute to different processes, depending on the exclusivity and the presence/absence of polarization degrees of freedom; in this work we will not be especially concerned with polarized targets. In particular, for the case of inclusive, unpolarized target, parity–conserving scattering only the transverse (T) and longitudinal (L) response functions contribute,

while for inclusive parity–violating scattering (necessitating a polarized electron beam) one also gains access to a T' response [9, 23]. The TL, TT and TL' responses enter in coincidence processes and when hadronic polarizations are involved [20, 21]. Explicit formulae in the general case for the kinematical factors v_i and the definition of the responses R^i_{fi} in terms of components of the hadronic tensor can be found in those references.

One is now confronted with the problem of calculating the hadronic tensor in Eq. (3). As stated in the Introduction, in this work we wish to extend the PWIA in a direction that incorporates specific interference effects. We therefore begin by reviewing the approximations that lead to the factorized PWIA. In so–doing we draw upon the discussion in Ref. [15]. Specifically, the PWIA contains three approximations: one–body current operators, plane–wave final state (hence, no FSI), and the assumption that the detected nucleon is the one that reacted with the virtual photon [24]. Under the first assumption one has for the electromagnetic current operators [14]

$$\hat{J}_\mu = \sum_{m,m'} \sum_{\tau,\tau'} \int d\boldsymbol{u} \int d\boldsymbol{u}' \langle \boldsymbol{u}', m', \tau' | \hat{j}_\mu | \boldsymbol{u}, m, \tau \rangle a^\dagger_{\boldsymbol{u}'m'\tau'} a_{\boldsymbol{u}m\tau} \; , \tag{7}$$

where $|\boldsymbol{u}, m, \tau\rangle$ is an on–shell nucleon spinor with momentum $\boldsymbol{u}$ and spin projection $m = \pm\frac{1}{2}$ referring to an axis of quantization along $\boldsymbol{q}$, unless otherwise specified. The label $\tau = \pm\frac{1}{2}$ denotes the isospin of the nucleon. The one–body operator $\hat{j}_\mu$ represents the physics of the γNN vertex and reads $\hat{j}_\mu = \exp(iQ_\alpha X^\alpha)\Gamma_\mu(Q^2)$. The vertex function $\Gamma_\mu(Q^2)$ contains the Dirac structure of the struck nucleon.

The second and third assumptions can be summarized in the formula

$$\langle f | a^\dagger_{\boldsymbol{p}'m'\tau'} = \delta(\boldsymbol{p}_N - \boldsymbol{p}')\delta_{m_N,m'}\delta_{\tau_N,\tau'}\langle B| \tag{8}$$

with the following interpretation: the final state $|f\rangle$ contains a plane–wave emitted nucleon plus the residual state $|B\rangle$, supplemented with the *additional assumption* that the emitted nucleon is the nucleon to which the virtual photon is connected. It is precisely this last assumption that we wish to relax in order to obtain the PWBA. Before doing so let us briefly see how the above three assumptions lead to the familiar factorized PWIA [15, 24]. Inserting Eq. (7) and using Eq. (8) to calculate $\langle f | \hat{J}_\mu | i \rangle$, we find

$$\langle f | \hat{J}_\mu | i \rangle = \sum_m \int d\boldsymbol{u} \langle \boldsymbol{p}_N, m_N | \hat{j}^\mu_{\tau_N} | \boldsymbol{u}, m \rangle \langle B | a_{\boldsymbol{u}m\tau_N} | i \rangle \; . \tag{9}$$

Upon squaring to obtain the hadronic tensor we find

$$W^{\mu\nu}(\boldsymbol{p}_N, \boldsymbol{q}) = \sum_{m,m'} \mathcal{W}^{\mu\nu}_{mm'}(\boldsymbol{p}, \boldsymbol{p}_N)\, \eta_{mm'}(\boldsymbol{p})\delta^3(\boldsymbol{p} + \boldsymbol{q} - \boldsymbol{p}_N) \; , \tag{10}$$

where we define the *single–nucleon tensor*

$$\mathcal{W}^{\mu\nu}_{mm'}(\boldsymbol{p}, \boldsymbol{p}_N) = \sum_{m_N} \langle \boldsymbol{p}_N, m_N | \Gamma^\mu | \boldsymbol{p}, m' \rangle^* \langle \boldsymbol{p}_N, m_N | \Gamma^\nu | \boldsymbol{p}, m \rangle \tag{11}$$

and the *momentum distribution*

$$\eta_{mm'}(\boldsymbol{p}) = \sum_B \langle B | a_{\boldsymbol{p}m'} | i \rangle^* \langle B | a_{\boldsymbol{p}m} | i \rangle \; , \tag{12}$$

4

where the spin content has been explicitly taken into account. It can be shown that when the target is unpolarized both the single–nucleon tensor and the momentum distribution are diagonal in the spin sector, that is $m = m'$ [15, 24]. Under these assumptions the electromagnetic interaction occurs only via the single–nucleon tensor which, apart from uncertainties due to the off–shellness of the struck nucleon $|\boldsymbol{p}, m\rangle$ [25], can be computed relativistically, as stated in the Introduction. A graphical depiction of the PWIA approximation is presented in Fig. 2a.

3. The Plane–Wave Born Approximation (PWBA)

We now relax the assumption that the detected nucleon is the same as the one that was struck by the virtual photon and restrict ourselves to a deuteron target. The physical interpretation of relaxing the above assumption in the case of the deuteron can be seen in Fig. 2. Specifically, suppose for the moment that one performs a coincidence ^{2}H$(e, e'\mathrm{p})$n measurement. While a proton is known to be detected, it might happen that it was either the proton (Fig. 2a) or the neutron (Fig. 2b) to which the photon connected. Some aspects of this possibility have been taken into account in the past (see for example [14] and references therein[1]), although one does not usually take these exchange effects into account at the same time treating the single–nucleon vertex to all orders in $1/M$. Indeed, the calculations of Arenhövel [18] and Fabian and Arenhövel [8] are the closest to our approach in that they retain terms up to order $1/M$ in the electromagnetic operators; here our goal is to incorporate specific relativistic ingredients to all orders while exploring such exchange effects and hence this aspect of our work is a natural extension of those previous studies. In addition, while these ideas could be applied to coincidence electron scattering, our present focus is on the roles that relativity and exchange play for inclusive processes (which are less commonly addressed in terms of exchange effects) and, consequently, after this section we shall specialize to the reactions ^{2}H(e, e')pn and ^{2}H$(\vec{e}, e')$pn, the latter involving parity–violating electron scattering.

We proceed by replacing Eq. (8) by

$$\langle f|a^{\dagger}_{\boldsymbol{p}',m',\tau'} = \delta^3(\boldsymbol{p}' - \boldsymbol{p}_N)\delta_{m',m_N}\delta_{\tau',\tau_B}\langle \boldsymbol{p}_B, m_B, \tau_B|$$
$$- \delta^3(\boldsymbol{p}' - \boldsymbol{p}_B)\delta_{m',m_B}\delta_{\tau',\tau_B}\langle \boldsymbol{p}_M, m_M, \tau_M| . \qquad (13)$$

Here the undetected nucleon in the deuteron is labelled "B" in order to connect with the conventions of Sec. 2. The minus sign in front of the second term is there to ensure antisymmetrization of the two–fermion state. Using the one–body *ansatz*, Eq. (7), we obtain for the electromagnetic matrix elements

$$\langle f|\hat{J}_{\mu}|d\rangle = \sum_m \sum_{\tau} \left\{ \int d\boldsymbol{u}\, \langle \boldsymbol{p}_N, m_N, \tau_N|\hat{j}_{\mu}|\boldsymbol{u}, m, \tau\rangle \langle \boldsymbol{p}_B, m_B, \tau_B|\, a_{\boldsymbol{u},m,\tau}\, |d\rangle \right.$$

---

[1]In a recent preprint Hummel and Tjon[26] have also gone further in addressing the issue of relativistic effects in parity–conserving deuteron electrodisintegration within a quasipotential framework where exchange (Born) terms and FSI are taken into account; in our work we focus on the former for both parity–conserving and parity–violating inclusive electrodisintegration, but do not consider the latter. The similarity of the results obtained when the same responses are considered reinforces our emphasis on the incorporation of the exchange effects via the relativistic PWBA. Note that the (more limited) approach taken in the present work can also be applied straightforwardly to nuclei with $A > 2$ [11].

$$-\int d\boldsymbol{u}\langle \boldsymbol{p}_B, m_B, \tau_B | \hat{j}_\mu | \boldsymbol{u}, m, \tau \rangle \langle \boldsymbol{p}_N, m_N, \tau_N | a_{\boldsymbol{u},m,\tau} | d \rangle \Big\} \ . \tag{14}$$

We now calculate the break–up amplitudes for a deuteron with the following quantum numbers: isospin $T = 0$ and $M_T = 0$, orbital angular momentum $L \in \{0,2\}$ coupled with spin $S = 1$ to total angular momentum $J = 1$ and prepared in a magnetic substate M_J:

$$\langle \boldsymbol{p}_1, m_1, \tau_1 | a_{\boldsymbol{p}_2,m_2,\tau_2} | J, M_J \rangle = \tag{15}$$

$$\mathcal{N} \, \delta^3(\boldsymbol{p}_1 + \boldsymbol{p}_2) \sum_{L=0,2} \sum_{M_L=-L}^{+L} i^L \, u_L(p_1) Y_L^{M_L}(\Omega_{\boldsymbol{p}_1})(-1)^{1+M_J+M_S} \times$$

$$\sum_{M_S=-S}^{+S} \begin{pmatrix} L & S & J \\ M_L & M_S & -M_J \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & S \\ m_1 & m_2 & -M_S \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & T \\ \tau_1 & \tau_2 & -M_T \end{pmatrix} \ .$$

Here $\mathcal{N}$ is an overall normalization factor and $u_L(k)$ the Bessel transforms of the S– and D–state radial wave functions of the deuteron (for $L = 0$ and $L = 2$ respectively). Inserting into Eq. (14) we obtain (using the fact that the single–nucleon matrix elements in Eq. (14) are diagonal in isospin space)

$$\langle f | J^\mu | d \rangle = \sum_m \Big\{ A_m^B \langle \boldsymbol{p}_N, m_N | \Gamma_{\tau_N}^\mu | \boldsymbol{p}, m \rangle + A_m^N \langle -\boldsymbol{p}, m_B | \Gamma_{\tau_B}^\mu | -\boldsymbol{p}_N, m \rangle \Big\} \ , \tag{16}$$

where we define for $i \in \{N, B\}$

$$A_m^i \equiv \frac{\mathcal{N}}{\sqrt{2}}(-1)^{1/2-\tau_N} \delta_{\tau_N,-\tau_B} \sum_L \sum_{M_L} \sum_{M_S} \begin{pmatrix} L & 1 & 1 \\ M_L & M_S & -M_J \end{pmatrix} \times$$

$$(-1)^{1+M_J+M_S} u_L(p_i) Y_L^{M_L}(\Omega_{\boldsymbol{p}_i}) \begin{pmatrix} 1/2 & 1/2 & 1 \\ m_i & m & -M_S \end{pmatrix} \ . \tag{17}$$

In the following we will freely use $\boldsymbol{p}_B + \boldsymbol{p} = 0$. The physics content of Eq. (16) can be made more transparent by examining Fig. 2.

The hadronic tensor for the unpolarized coincidence process where the nucleon $i = N$ is detected then reads (after we sum over the final–state quantum numbers and average over the initial–state ones):

$$W^{\mu\nu} = \frac{1}{2J+1} \sum_{M_J} \sum_{m_N} \sum_{\tau_B} \sum_{m_B} \sum_m \sum_{m'} \Big\{ D^{\mu\nu} + D'^{\mu\nu} + E^{\mu\nu} + E'^{\mu\nu} \Big\} \ , \tag{18}$$

where we have defined two "Direct" and two "Exchange" tensors as

$$\begin{aligned}
D^{\mu\nu} &\equiv A_{m'}^{B*} A_m^B \langle \boldsymbol{p}_N, m_N | \Gamma_{\tau_N}^\mu | -\boldsymbol{p}_B, m' \rangle^* \langle \boldsymbol{p}_N, m_N | \Gamma_{\tau_N}^\nu | -\boldsymbol{p}_B, m \rangle & (19) \\
D'^{\mu\nu} &\equiv A_{m'}^{N*} A_m^N \langle \boldsymbol{p}_B, m_B | \Gamma_{\tau_B}^\mu | -\boldsymbol{p}_N, m' \rangle^* \langle \boldsymbol{p}_B, m_B | \Gamma_{\tau_B}^\nu | -\boldsymbol{p}_N, m \rangle \\
E^{\mu\nu} &\equiv A_{m'}^{B*} A_m^N \langle \boldsymbol{p}_N, m_N | \Gamma_{\tau_N}^\mu | -\boldsymbol{p}_B, m' \rangle^* \langle \boldsymbol{p}_B, m_B | \Gamma_{\tau_B}^\nu | -\boldsymbol{p}_N, m \rangle \\
E'^{\mu\nu} &\equiv A_{m'}^{N*} A_m^B \langle \boldsymbol{p}_B, m_B | \Gamma_{\tau_B}^\mu | -\boldsymbol{p}_N, m' \rangle^* \langle \boldsymbol{p}_N, m_N | \Gamma_{\tau_N}^\nu | -\boldsymbol{p}_B, m \rangle \ .
\end{aligned}$$

A diagrammatical interpretation of these tensors is given in Fig. 3. The direct contributions amount to incoherent scattering, whereas the two exchange contributions incorporate the interference terms. It is important to notice that the *coincidence* PWIA corresponds to just one of the direct terms, namely $D^{\mu\nu}$. We first obtain

$$
\begin{aligned}
\sum_{\tau_B}\sum_{M_J} A_{m'}^{j*} A_m^i &= \frac{1}{2}\mathcal{N}^2 \sum_{L,L'} \sum_{M_L,M_L'} \sum_{M_S,M_S'} \sum_{K=0,1,2} \sum_{X=-K}^{+K} (2K+1)(-1)^{M_L+M_S+1} \\
&\times \begin{pmatrix} 1 & 1 & K \\ M_S & -M_S' & X \end{pmatrix} u_L(p_i) u_{L'}(p_j) \begin{Bmatrix} L & L' & K \\ 1 & 1 & 1 \end{Bmatrix} Y_L^{M_L}(\Omega_{\boldsymbol{p}_i}) Y_{L'}^{*M_L'}(\Omega_{\boldsymbol{p}_j}) \\
&\times \begin{pmatrix} L & L' & K \\ M_L & -M_L' & -X \end{pmatrix} \begin{pmatrix} 1/2 & 1/2 & 1 \\ m_i & m & -M_S \end{pmatrix} \begin{pmatrix} 1/2 & 1/2 & 1 \\ m_j & m' & -M_S' \end{pmatrix} .
\end{aligned} \tag{20}
$$

Let us calculate the direct terms $(i = j)$. It can be shown that only $K = 0$ contributes in this case. This also forces diagonality in spin space, $m = m'$, and one is left with the expression

$$
\begin{aligned}
\sum_{spins} D^{\mu\nu} &= \frac{\mathcal{N}^2}{24\pi(2J+1)} \sum_L |u_L(p)|^2 \sum_{m_N} \sum_{m,m'} \delta_{m,m'} \\
&\times \mathrm{Tr}\left\{|\boldsymbol{p},m\rangle\langle\boldsymbol{p},m'|\gamma^0\Gamma_{\tau_N}^{\mu\dagger}\gamma^0|\boldsymbol{p}_N,m_N\rangle\langle\boldsymbol{p}_N,m_N|\Gamma_{\tau_N}^\nu\right\} ,
\end{aligned} \tag{21}
$$

for the PWIA term and a similar one for the other direct term $D'^{\mu\nu}$ which, however, does not contribute to the coincidence PWIA, since if D is being calculated for $(e, e'p)$ then D' corresponds to $(e, e'n)$ and *vice versa.* Note that the spin sums decouple between the single–nucleon matrix elements and the break–up amplitude $3 - j$ symbols. The PWIA answer for the single–nucleon tensor corresponding to the reaction $^2\mathrm{H}(e, e'\mathrm{N})\mathrm{B}$ is then simply

$$
\mathcal{W}_{PWIA}^{\mu\nu} = \frac{\mathcal{N}^2}{24\pi(2J+1)(2M)^2} \mathrm{Tr}\left\{[\not{p} + M]\gamma^0\Gamma_{\tau_N}^{\mu\dagger}\gamma^0[\not{p}_N + M]\Gamma_{\tau_N}^\nu\right\} , \tag{22}
$$

where comparison with Eq. (10) shows that the momentum distribution in this case is simply $\eta(p) \sim \sum_L u_L(p)^2$.

The situation is more complicated in the case of the exchange terms for two reasons. Firstly, note from Eq. (19) that both m_N and m_B appear in the single–nucleon parts; moreover, the arguments in the spherical harmonics are not identical any more. As a result, the spin sums cannot be done without the explicit spin–dependence of the single–nucleon matrix elements. In general, all K–values contribute and m, m' need not be equal as before. Secondly, the spinor–product sums one encounters are not any longer diagonal in momentum space (for example, one needs $|\boldsymbol{p}_B, m\rangle\langle\boldsymbol{p}_N, m'|$). In order to overcome the non–diagonality in the spin sector we could proceed as in Ref. [15] where we introduced a diagonal hadronic tensor with spinors quantized with respect to a generic direction. However, to take care of the non–diagonality in momentum space we would have to boost the $|\boldsymbol{p}_B, m\rangle$ state into, say, one with momentum $\boldsymbol{p}_N$. Together with the γ_5's that are parts of the spin–projectors needed (due to the decoupling of the spin sums) to convert the single–nucleon part into a trace, this would lead to a trace with a large number of γ matrices. For these reasons we have abandoned the idea of a trace formulation and, instead, we evaluate the spin sums at the outset. To that end we will use the notation of

Alberico *et al.* in Ref. [27] and decompose the single–nucleon operators into spin–flip (s) and non–spin–flip (n) parts:

$$J^\mu_{m_N,m_i} \equiv \langle \boldsymbol{p}_N, m_N | \Gamma^\mu_{\tau_N} | -\boldsymbol{p}_B, m_i \rangle = J^\mu_{(n)}\delta_{m_N,m_i} + J^\mu_{(s)}(2m_i)(1-\delta_{m_N,m_i})$$
$$H^\mu_{m_B,m_i} \equiv \langle \boldsymbol{p}_B, m_B | \Gamma^\mu_{\tau_B} | -\boldsymbol{p}_N, m_i \rangle = H^\mu_{(n)}\delta_{m_B,m_i} + H^\mu_{(s)}(2m_i)(1-\delta_{m_B,m_i}) \ . \tag{23}$$

For the 3–vector currents in the above equation we choose a coordinate system with the 3–axis along the momentum transfer $\boldsymbol{q}$, the 2–axis along $\boldsymbol{v} = \boldsymbol{q} \times \boldsymbol{p}$ (that is, perpendicular to the hadron plane) and the 1–axis along $\boldsymbol{v} \times \boldsymbol{q}$ (in the hadron plane).

$$\boldsymbol{J}_{m_j,m_i} = J^{(1)}_{m_j,m_i}\hat{\boldsymbol{u}}_1 + (2m_i)J^{(2)}_{m_j,m_i}\hat{\boldsymbol{u}}_2 + J^{(3)}_{m_j,m_i}\hat{\boldsymbol{u}}_3 \ , \tag{24}$$

and a similar equation for $\boldsymbol{H}_{m_j,m_i}$. Detailed expressions for the matrix elements in Eq. (23) in terms of the single–nucleon form factors can be found in the Appendix. We can now perform all the spin sums for both the direct– and exchange–type terms. The sums in the former case are trivial (as expected) and one ends up with the following formula for the direct term, which should be compared with Eq. (21):

$$\sum_{spins} D^{\mu\nu} = \begin{cases} 0 \ , & \text{if } \mu = 2 \text{ or } \nu = 2 \quad \text{, but not both} \\ \frac{\mathcal{N}^2}{72\pi}\sum_L |u_L(p_B)|^2 \left\{ J^{*\mu}_{(n)}J^\nu_{(n)} + J^{*\mu}_{(s)}J^\nu_{(s)} \right\} & \text{, otherwise} \end{cases} \tag{25}$$

and a similar expression for the other direct term

$$\sum_{spins} D'^{\mu\nu} = \begin{cases} 0 \ , & \text{if } \mu = 2 \text{ or } \nu = 2 \quad \text{, but not both} \\ \frac{\mathcal{N}^2}{72\pi}\sum_L |u_L(p_N)|^2 \left\{ H^{*\mu}_{(n)}H^\nu_{(n)} + H^{*\mu}_{(s)}H^\nu_{(s)} \right\} & \text{, otherwise.} \end{cases} \tag{26}$$

We finally proceed to obtain the two exchange terms. After evaluating the spin summations we can cast both exchange contributions in the compact form

$$\sum_{spins} [E^{\mu\nu} + E'^{\mu\nu}] = \mathcal{E}^{\mu\nu} + \mathcal{E}^{*\nu\mu} \ , \tag{27}$$

where we have set

$$\mathcal{E}^{\mu\nu} = \frac{1}{6}\mathcal{N}^2 \sum_K \sum_X \sum_{L,L'} u_L(p)u_{L'}(p_N) \sum_{M_L,M'_L} \begin{Bmatrix} L & L' & K \\ 1 & 1 & 1 \end{Bmatrix} (2K+1)(-1)^{M_L}$$
$$\times Y^{M_L}_L(\Omega_{\boldsymbol{p}})Y^{*M'_L}_{L'}(\Omega_{\boldsymbol{p}_N}) \begin{pmatrix} L & L' & K \\ M_L & -M'_L & -X \end{pmatrix}$$
$$\times \sum_{i,j\in\{n,s\}} a^{K,X}_{i,j} H^{*\mu}_{(i)}J^\nu_{(j)} \ . \tag{28}$$

The coefficients $a^{K,X}_{i,j}$ are defined in the Appendix and appear in Table I.

4. The S–State–Only, Nonrelativistic, Inclusive PWBA

Eq. (28) cannot be further simplified unless certain approximations are made. In the next–to–leading order nonrelativistic limit it reproduces the results of Arenhövel (Eqs. (A.1)–(A.5) in Ref. [18]), notwithstanding the fact that our calculation is performed in the laboratory frame whereas the one in Ref. [18] is performed in the center–of–mass frame. Our aim here is to gain some understanding of the difference between the PWBA and PWIA in the *inclusive* case. The inclusive PWIA is defined to be the sum of the individual contributions obtained by integrating over final momenta in the ^{2}H(e,e'p)n and ^{2}H(e,e'n)p cross sections. In other words, the inclusive PWIA amounts to the *total incoherent* part of the PWBA, unlike the coincidence situation, where the $D'^{\mu\nu}$ term in Eq. (19), although belonging to the incoherent contribution, is *not* part of the coincidence PWIA.

We first wish to obtain a qualitative understanding of the role of interference terms in the inclusive electrodisintegration of deuterium. To that end here we shall make several approximations; of course, these approximations are not made when presenting results in later sections. Referring to Eq. (28) and Table I we first note that restricting ourselves to the S–state–only yields considerable simplification as then $K = X = 0$ and also all angular dependence becomes trivial. It should be noted that this is not an unreasonable approximation for *quasielastic*, inclusive electron scattering. The reason is that the strength of the inclusive QE response comes predominantly from the low momentum components of the struck nucleon [22], whereas the D–state contribution (in momentum space) becomes comparable to the S–state–only around 250 MeV/c, more than 4 times the deuteron Fermi momentum of about 55 MeV/c (which is manifested in the width of the QE response [28]). The form of the PWBA longitudinal and transverse responses in the S–state–only limit is given in the Appendix. We now make a further simplification in this initial look at the responses and restrict ourselves to the leading nonrelativistic contributions for $J^{\mu}_{(i)}$, $H^{\mu}_{(i)}$, which as seen in Table II involve the electric and magnetic electromagnetic single–nucleon form factors $G^{\tau}_{E,M}$, where $\tau = \tau_N$ or τ_B labels the isospin projection (see Ref. [27]). The "3" components are eliminated by current conservation [21] and we define $\kappa \equiv q/2M$.

In the coordinate system we have chosen, the longitudinal and transverse responses are given in terms of the following components of the hadronic tensor [15]: $R_L = W^{00}$ and $R_T = W^{11} + W^{22}$. Referring now to Eqs. (61,62) in the Appendix we can write for the longitudinal response in the nonrelativistic, S–state–only limit:

$$R_L = \frac{\mathcal{N}^2}{72\pi} \left\{ u^2(p_B) G_E^{\tau_N\,2} + u^2(p_N) G_E^{\tau_B\,2} + 2u(p_N)u(p_B) G_E^{\tau_N} G_E^{\tau_B} \right\} . \tag{29}$$

Similarly, for the transverse response we have

$$R_T = \frac{\mathcal{N}^2}{72\pi} \times \kappa^2 \left\{ u^2(p_B) G_M^{\tau_N\,2} + u^2(p_N) G_M^{\tau_B\,2} + \left(\frac{2}{3}\right) u(p_N)u(p_B) G_M^{\tau_N} G_M^{\tau_B} \right\} . \tag{30}$$

The same nonrelativistic limits have been obtained with a simple calculation in Refs. [9, 29]. In order to obtain the inclusive EM responses, we integrate over final momenta including the energy–conserving delta function $\delta(\omega - \Omega)$ where

$$\Omega(\boldsymbol{p}_N, \boldsymbol{p}_B) = \sqrt{\boldsymbol{p}_N{}^2 + M^2} + \sqrt{\boldsymbol{p}_B{}^2 + M^2} - M_d = \Omega(\boldsymbol{p}_B, \boldsymbol{p}_N) . \tag{31}$$

We define the following "Direct" and "Exchange" integrals $\mathcal{D}$ and $\mathcal{E}$

$$\mathcal{D} = \int d\boldsymbol{p}\delta(\omega - \Omega)u^2(p)$$

$$\mathcal{D}' = \int d\boldsymbol{p}\delta(\omega - \Omega)u^2(p_N) = \mathcal{D}$$

$$\mathcal{E} = \int d\boldsymbol{p}\delta(\omega - \Omega)u(p)u(p_N) , \tag{32}$$

where as always $\boldsymbol{p}_B + \boldsymbol{p}_N = 0$ and Eq. (31) guarantees that $\mathcal{D} = \mathcal{D}'$. Then for the inclusive responses we have

$$R_L \sim \left\{ G_E^{T=0^2}\left(\mathcal{D}+\mathcal{E}\right) + G_E^{T=1^2}\left(\mathcal{D}-\mathcal{E}\right) \right\}$$

$$R_T \sim \kappa^2 \left\{ G_M^{T=0^2}\left(\mathcal{D}+\frac{\mathcal{E}}{3}\right) + G_M^{T=1^2}\left(\mathcal{D}-\frac{\mathcal{E}}{3}\right) \right\} , \tag{33}$$

where we have omitted an overall constant. The factor of 3 in the exchange part of the transverse responses is a consequence of the spin–flip character of the relevant nonrelativistic operators [29] and the fact that the spin of the deuteron is 1. The importance of this factor in suppressing the role of the exchange terms has been also observed by Frankfurt and Strikman in a light cone calculation [30]. Comparing the above expressions with the inclusive PWIA, we note that the latter is defined as the sum of the integrals over final momenta of the ${}^2\mathrm{H}(e,e'p)n$ and ${}^2\mathrm{H}(e,e'n)p$ cross sections, and therefore reproduces just the $\mathcal{D}$ terms of Eq. (33).

4.1. The $\mathcal{E}/\mathcal{D}$ Ratio

This suggests that the $\mathcal{E}/\mathcal{D}$ ratio sets the naïve scale for the importance of the interference terms. We say naïve, because the actual scale for the various response functions can be very different from $\mathcal{E}/\mathcal{D}$, due to the specifics of the form factors involved, as we shall discuss shortly. To begin with we examine the behaviour of this ratio as a function of the kinematical variables q and ω. Let us first perform the angular integrations in Eq. (32) using the energy–conserving delta function, and rewrite the Direct and Exchange integrals in the form

$$\mathcal{D} = 2\pi \int_{|y(q,\omega)|}^{Y(q,\omega)} pdp\frac{E_N(p)}{q}u^2(p)$$

$$\mathcal{E} = 2\pi \int_{|y(q,\omega)|}^{Y(q,\omega)} pdp\frac{E_N(p)}{q}u(p)u(p_N) , \tag{34}$$

where

$$E_N(p) = \sqrt{p_N^2 + M^2} = M_d + \omega - \sqrt{p^2 + M^2} . \tag{35}$$

The transition from Eqs. (32) to Eqs. (34), as well as the limits of integration are discussed in Refs. [22, 29]. There are two questions we wish to address: firstly, what is the behaviour of $\mathcal{E}/\mathcal{D}$ with ω for fixed momentum transfer q, and secondly, what is the behaviour of $\mathcal{E}/\mathcal{D}$ as a function of the momentum transfer q for fixed QE kinematics, $\omega \approx |Q^2|/2M$? Strictly speaking, the range of ω values for which we can legitimately address the first question in the context of a plane–wave, no MEC model, is restricted to the regime of quasifree scattering as characterized by the width of the QE peak, $\Delta\omega = \sqrt{2}qp_F/\sqrt{M^2 + q^2}$ [9].

We first plot in Fig. 4 the integrands of the Direct and Exchange contributions for a small value of momentum transfer, $q = 150$ MeV/c. Here we plot $pu^2(p)$ and $pu(p)u(p_N)$, which are, respectively, the integrands of $\mathcal{D}$ and $\mathcal{E}$ in the nonrelativistic limit (up to a constant), where $u(p)$ is the Bessel transform of the S–state deuteron wave function. It is clear that, being linear in p, the direct integrand starts from zero at the origin, then peaks due to the fact that $u(p)$ drops dramatically with p, and eventually dies off with larger values of p. On the other hand, the form of the exchange integrand, $\mathcal{E}$, for fixed momentum transfer q, depends on the value of the energy transfer ω, since for each given momentum p we must solve Eq. (35) in order to find the corresponding value of p_N. We show this integrand for two characteristic values of ω, namely, one close to threshold, where $\omega_{thr} + M_d = \sqrt{(AM)^2 + q^2}$ with $A = 2$ for the deuteron, and one on the quasielastic peak, where $\omega_{qe} + M_d = \sqrt{M^2 + q^2} + M$ [22]. To understand the form of the integrands, we need to know approximately where $p_N(q, \omega, p) = p$, since this will give us the point where the $\mathcal{E}$ and $\mathcal{D}$ integrands intersect. In this particular case $M \gg q$, since $q = 150$ MeV/c and $M \approx 939$ MeV, and ignoring the small (≈ 2.25 MeV) deuteron binding energy, we can solve Eq. (35) at threshold as

$$p^2 + p_N^2 = \frac{q^2}{2} \,, \tag{36}$$

that is, $p = p_N$ at $p = q/2$, which means that the two integrands cross around $p = q/2 = 75$ MeV/c, as indeed is seen in the Fig. 4a. On the other hand, the point where $p = p_N$ in the case where $\omega = \omega_{qe}$ is found from Eq. (35) as

$$p^2 + p_N^2 = q^2 \,, \tag{37}$$

that is, $p = p_N$ at $p = q/\sqrt{2}$, which is approximately at $p = 107$ MeV/c, as seen in the Fig. 4b. As far as the limits of integration are concerned, in the case of the deuteron they are given by [29]

$$p_{min} = |y|, \qquad \text{where } y = (M_d + \omega)\sqrt{\frac{1}{4} - \frac{M^2}{W^2}} - \frac{q}{2}$$
$$p_{max} = Y, \qquad \text{where } Y = y + q \,, \tag{38}$$

with $W^2 = (M_d + \omega)^2 - q^2$ being the center–of–mass energy. On the peak, $y = 0$ and therefore the limits extend from 0 to q [22]. Since the two integrands cross at 107 MeV/c (a high value compared to the Fermi momentum $p_F = 55$ MeV/c, which characterizes the falloff of $u(p)$) and both start from zero, we learn two things: first, that $\mathcal{D}$ will be a maximum on the quasielastic peak, (hence the very notion of the *peak* for $\omega = \omega_{qe}$), and second that the $\mathcal{E}/\mathcal{D}$ ratio on the peak will be rather small. This is illustrated in Fig. 4b, where we have graphically depicted the two integrals by shading the corresponding integrands.

However the situation is entirely different near threshold. There, the limits of integration are $p_{min} = p_{max} = q/2$, which means that as we approach threshold from above, the two integrals vanish; what is interesting, however, is that their *ratio* approaches 1. This happens because it is precisely at $p = q/2$ that the two integrands are equal, as we argued above. That means that for ω close to threshold, we will be integrating in an area just around $p = q/2$, where by continuity the two integrands will be very similar and therefore the $\mathcal{E}/\mathcal{D}$ ratio will be close to 1. We depict

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this situation in Fig. 4a for $\omega = 9$ MeV, where the limits are $p_{min} = 47$ MeV/c and $p_{max} = 102$ MeV/c. We can clearly see that the two shaded areas, corresponding to the two integrals, are very similar.

We summarize this behaviour of the $\mathcal{E}/\mathcal{D}$ ratio as a function of energy transfer, for two cases $q = 150$ MeV/c and $q = 300$ MeV/c in Figs. 5a and 5b. In order to provide a feeling for the individual integrals, and not just their ratio, we have included Tables III.1 and III.2. The position of the QE peak for those q values is at 14.13 and 49.89 MeV, respectively.

Let us next examine the behaviour of the $\mathcal{E}/\mathcal{D}$ ratio as a function of the momentum transfer q, on the quasielastic peak. It is precisely around the quasielastic peak that we hope to be able to trust a quasifree calculation like the plane–wave approximation, and an understanding of the role of the interference terms as given by the Exchange integral in that kinematical region therefore seems necessary. We begin by noticing that the larger the value of momentum transfer, the further apart are p and $p_N(p, \omega_{qe}(q))$ located. To see why this happens, let us recall that in terms of the y–scaling variable we have [9]

$$M_d + \omega = E_B + E_N = \sqrt{M^2 + (q+y)^2} + \sqrt{M^2 + y^2} \ . \tag{39}$$

Examining the direct term we see that the integral has its maximum when $y = 0$ and thus $\omega = \omega_{qe}$. For fixed $y = 0$ the answer is independent of q, since the only other q–dependence in this simplified model is contained in Y and becomes negligible when q and $Y \to \infty$. On the other hand, the exchange term involves an overlap between $u(p)$ and $u(p_N)$. At $y = 0$ we have $E_B + E_N = \sqrt{M^2 + q^2} + M$, and therefore, as q becomes large so does the sum of $E_B = \sqrt{p^2 + M^2}$ and $E_N = \sqrt{p_N{}^2 + M^2}$. Hence, not both of p and p_N can be small at the same time. Since the function $u(p)$ is localized to very small values of p (i.e., most of the momentum distribution lies at $p < p_F \sim 55$ MeV/c), this immediately implies that the ratio $\mathcal{E}/\mathcal{D}$ must fall with increasing q. This is to be expected, since the importance of the interference terms scales roughly like q/p_F. The ratio $\mathcal{E}/\mathcal{D}$, which sets the naïve scale for the relative importance of the exchange terms, is shown in Fig. 6 for kinematics corresponding to the impulse approximation position of the QE peak,

$$\omega_{qe} = \sqrt{M^2 + q^2} + \epsilon_d - M \ , \tag{40}$$

where $\epsilon_d = 2M - M_d$ is the deuteron binding energy The characteristic momentum where this ratio becomes $1/2$ is about 110 MeV/c $\sim 2p_F$. This is suggestive of a simple Fermi gas picture where two momentum distributions with characteristic width p_F overlap considerably only when they are centered at momenta differing by less than $2p_F$. We see that the ratio drops dramatically with increasing momentum transfer and thus we should expect that the exchange contributions for deuterium become unimportant beyond, say, 400–500 MeV/c and accordingly that the assumption of ignoring the interference terms of the PWBA will become reasonable at sufficiently high q. However, at these high momentum transfer values we need to include the effects of the D–state and relativity we have ignored so far. This we will discuss in Sec. 5.

4.2. The role of form factors

Our previous analysis suggests that, at least around the quasielastic peak and at relatively high momentum transfer values, the differences between the de–relativized PWIA and PWBA should disappear. However, caution should be exercised [9, 29] even when studying only the region

near the QE peak, since the significance of the ratio $\mathcal{E}/\mathcal{D}$ may be quite different for the various response functions and, ultimately, for the parity–conserving cross section and parity–violating asymmetry, depending on the choice of kinematics.

Consider first the longitudinal EM response: for small momentum transfers, $G_E^{(T=0)} \simeq G_E^{(T=1)}$, since G_{En} is very small. Now we see from Eq. (33) that it is $\left(G_E^{(T=0)^2} + G_E^{(T=1)^2} \right)$ that multiplies $\mathcal{D}$, and $\left(G_E^{(T=0)^2} - G_E^{(T=1)^2} \right)$ that multiplies $\mathcal{E}$. As we have seen, $\mathcal{E}/\mathcal{D}$ is appreciable at small q values, precisely where the form factor combination that multiplies it is negligible, and therefore $R_L^{(T=0)} + R_L^{(T=1)}$ is basically the same whether the PWBA or PWIA is used. When G_{En} becomes appreciable, $\mathcal{E}/\mathcal{D}$ has already dropped dramatically, and thus the conclusion still holds. On the other hand, a different argument is found for the transverse EM terms, where $G_M^{(T=0)}$ and $G_M^{(T=1)}$ are rather different (equivalently, G_{Mp} and G_{Mn} are both large and different). As we can see from Eqs. (33), there the scale is given by $\mathcal{E}/3\mathcal{D}$, which suppresses the exchange effects, and hence we see that the PWBA and PWIA results should not be much different as long as $\mathcal{E}/\mathcal{D}$ is not too large. Similar conclusions hold for the transverse PV responses which are also spin–flip–dominated.

However, the results obtained for the *electroweak* response functions that appear in the numerator of the parity–violating asymmetry [9] in the process $^2\mathrm{H}(\vec{e}, e')\mathrm{np}$ may be very different. In particular, the electroweak longitudinal response $\tilde{R}^L$, whose contribution in the PV asymmetry is most important at forward angles, can be very sensitive to $\mathcal{E}/\mathcal{D}$ effects. This is due to the particular form of the weak neutral current couplings $\beta_V^{(T=0,1)}$ in terms of which the electroweak form factors become

$$
\begin{aligned}
\tilde{G}_{E,M}^{T=0} &= \beta_V^{(0)} G_{E,M}^{T=0} + \beta_V^{(s)} G_{E,M}^{(s)} \\
\tilde{G}_{E,M}^{T=1} &= \beta_V^{(1)} G_{E,M}^{T=0} \; ,
\end{aligned} \tag{41}
$$

where $\beta_V^{(1)} = 1 - 2\sin^2\theta_W$, $\beta_V^{(0)} = -2\sin^2\theta_W$ and $\beta_V^{(s)} = 1$ [31]. Ignoring the strangeness contributions (labelled "(s)") we see from Eq. (33) that the scale dictating the importance of the interference terms for the PV longitudinal response is changed from the naïve value $\mathcal{E}/\mathcal{D}$ to

$$
\left(\frac{\beta_V^{(0)} - \beta_V^{(1)}}{\beta_V^{(0)} + \beta_V^{(1)}} \right) \frac{\mathcal{E}}{\mathcal{D}} = \frac{-1}{1 - 4\sin^2\theta_W} \frac{\mathcal{E}}{\mathcal{D}} \; , \tag{42}
$$

which, due to the rather interesting coincidence that $\sin^2\theta_W \sim 0.227$ (*i.e.*, nearly 1/4 where the factor $1 - 4\sin^2\theta_W$ goes to zero) is about $-11\mathcal{E}/\mathcal{D}$ in the standard model [9]. For this reason, the parity–violating longitudinal response can be extremely different in the PWIA and PWBA, even leading to opposite signs for the asymmetry obtained near threshold using the two models, as we have demonstrated in Ref. [9]. This is illustrated in Fig. 7 where we plot the PV asymmetry for forward scattering ($\theta_e = 35°$) at $q = 150$ and 300 MeV/c using both the PWIA and PWBA models: whereas the results for the cross section are indistinguishable (not shown), the PV asymmetries are extremely different for the two approximations, especially as one goes towards threshold. For more discussion of the PV asymmetry, see Refs. [9, 31].

5. The Role of Relativity

In this section we present results for kinematics not restricted to the quasielastic region. For such kinematics important corrections to the plane–wave approximations such as final–state interactions, meson exchange currents and pion production effects can be important. In the present work our focus is not on such aspects of the full problem, but rather on a specific model where *both relativistic and exchange* effects can be incorporated at the same time. To that end we define the following ratios for subsequent use, involving the fully–relativistic Born (BF) and Impulse (IF) models, as well as a fully–relativistic S–state–only version (BSF and ISF) and the nonrelativistic S–state–only Born and Impulse reductions discussed in the previous section (BN and IN, respectively):

$$\Delta_{BF/IF} \equiv |BF/IF - 1| \; , \tag{43}$$

which is a measure of the difference between the Born and Impulse approximation, and

$$\Delta_{BSF/BN} \equiv |BSF/BN - 1| \; , \tag{44}$$

which is a measure of the importance of the relativistic corrections.

We start our presentation of the full results with the absolute value of the ratio $\mathcal{E}/\mathcal{D}$ which sets the naïve scale for the importance of exchange effects. In Fig. 8 we plot percentage contours for this ratio, for $100 \leq q \leq 800$ MeV/c and energy transfer ω from threshold up to $max\{q, 400$ MeV$\}$. In all of the following contour plots we indicate the position of the quasielastic ridge. One observes that the ratio has a trough at low q which coincides with the quasielastic ridge and a clear ridge near threshold, as anticipated from the discussion in the previous section and Fig. 4. Thus, the $\mathcal{E}/\mathcal{D}$ ratio for quasifree kinematics is less than 10% for q above 250–300 MeV/c. Moving away from the quasifree region we observe another ridge for large values of energy transfer near the real–photon line. This maximum is due to the specifics of the deuteron wave functions used (Reid soft core) and is not reproduced by simplistic wave functions (see next section).

Next we discuss the $\Delta_{BF/IF}$ ratio for the two responses accessible in unpolarized EM inclusive scattering, R_L and R_T. In the following we use the Galster parametrization for the nucleon form factors [32]. The R_L results are presented in Fig. 9. It is clear that unless one stays very close to threshold, the longitudinal response is almost totally oblivious to the exchange terms. The only remnant of the second ridge in the $\mathcal{E}/\mathcal{D}$ ratio in Fig. 8 is a 1% effect, which, however, lies outside of the quasifree region. Thus our qualitative analysis of Sec. 4 tracing the insensitivity of R_L to the exchange effects back to the fact that G_{En} is negligible at those values of Q^2 where $\mathcal{E}/\mathcal{D}$ is appreciable proves to be adequate even for the full calculation. This at first sight seems strange, since the relativistic single–nucleon matrix elements now also have G_M contributions. Referring to the Appendix, however, we see that the dominant contribution of that type will be $G_{Ep}G_{Mn} \sinh \Phi \sin \Psi \cosh \Phi \cos \Psi$, which is proportional to $(p/M)^2$ and hence very much suppressed. For example, with a Fermi momentum distribution $\theta(p_F - p)$, for $q > p_F$ the contribution of the G_E terms to the $\Delta_{BF/IF}$ ratio for quasielastic kinematics is proportional to $1-(a/p_F)^2$, with $a = \sqrt{q^2 - p_F^2}$, whereas the contribution of the G_M terms would be proportional to $1 - (a/p_F)^4$.

In Fig. 10 we present the results for R_T. Again we observe that, except for very low values of momentum transfer the quasielastic ridge is less than 2% sensitive to exchange effects. However, this changes when one moves away from quasifree kinematics, especially towards threshold, and above the quasielastic peak around $q = 400$ MeV/c, where there is a 10% peak; the exchange terms there are definitely not negligible compared to other effects such as FSI[2] or MEC that are usually pointed to as shortcomings of the PWIA. We see that the magnitude of the effect is not the same as the $\mathcal{E}/\mathcal{D}$ ratio, the reason being that — unlike the longitudinal, where the suppression of exchange effects comes from the form factors — the leading interference contributions are suppressed by a factor of 3 compared to the direct ones (see Eq. (33)).

We now shift our attention to the importance of the relativistic corrections that we have included in the single–nucleon matrix elements. Looking at the explicit expressions of these matrix elements in the Appendix, we see that as a general rule such relativistic corrections occur in two classes: the first class involves terms like $(1 + \tau)$ (the Darwin term) or like $q^2/|Q^2|$. The second class involves terms that are proportional to $\sinh \Phi$ or $\sin \Psi$ and therefore proportional to $(p \sin \theta / M)$. These last terms are suppressed for two reasons: firstly, the Fermi momentum provides a cutoff for p which is especially low in the case of the deuteron; secondly, on the quasielastic peak we have $\omega = 2M\tau + 2M - M_d$, and therefore $p \sin \theta / M = \sqrt{\frac{|Q^2|}{q^2}(\frac{M_d + \omega}{2M})^2 - (1 + \tau)}$ vanishes. Accordingly, we expect the contribution of second–class terms to be small in the quasifree region where the dominant contribution to the momentum integrals comes from $p \sin \theta$ values close to 0 (this corresponds to the common — yet quite misleading — statement that quasifree kinematics amounts to the struck nucleon being at rest). However, the first–class terms can be very important with high values of momentum transfer. For example at the quasielastic peak corresponding to $q = 1$ GeV/c the factor $q^2/|Q^2|$ induces a 20% effect. Thus we see that there is indeed a simple reason why in the quasielastic region the two scales that drive the relativistic corrections to the problem, namely q/M and p/M, decouple: the latter is always associated with a $\sin \theta$ factor which vanishes on the quasielastic peak and is in general small around it. Note that, if we simply take $1/M$ as the "one" scale that characterizes the relativistic effects without being careful about the momentum that multiplies it, we would argue that it is the first–class terms that are of order $O[1/M]^2$ and thus have to be dismissed, while the second–class terms, being of order $O[1/M]$ have to be retained. For example, the calculation of Ref. [18] includes corrections up to order $[1/M]$ and thus ignores the first–class corrections while retaining terms linear in $(p \sin \theta / M)$. This is of course *consistent* with the treatment of the deuteron wave function in that work. *Numerically,* however, we wish to point out that the incorporation of the first–class–type corrections can be important. The above remarks support the comments made in the Introduction where we argue that in the quasifree region the "dynamical" relativistic corrections associated with the momentum of the struck nucleon decouple from the more "kinematical" factors associated with the large momentum transfer, like $q^2/|Q^2|$. Note that were we to perform our calculation in the center–of–mass frame we would lose the advantage of having the Fermi momentum cutoff as the only scale regarding relativistic corrections for the deuteron wave function and the struck nucleon, as now a q–dependent operator would be required in order to boost to the center–of–mass frame. Although the relativistic corrections induced by this boost can be shown in general not to be as significant as those induced by using

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[2]See also the footnote at the beginning of Sec. 3 where we point to the recent preprint of Hummel and Tjon[26].

relativistic single–nucleon current operators [3], we prefer to minimize them by working directly in the deuteron rest frame.

In Fig. 11a we show contour plots for the $\Delta_{BSF/BN}$ ratio (as defined above in Eq. (44)) in the case of the longitudinal response. Effects of order 10% are present in the quasifree region for q beyond 600 MeV/c, whereas for higher momentum transfer values there can be 10% effects already at 400 MeV/c and 20% effects at $q = 700$ MeV/c. We observe a ridge parallel to the quasielastic region but at momentum transfer values greater by about 150 MeV/c. This ridge is directly related to the G_M terms in the longitudinal EM response. To see that, first recall from Fig. 9 that for this response the exchange effects are less than 1% (except near threshold) and thus decouple from the relativistic effects, allowing the ratio $\Delta_{BSF/BN}$ to be analyzed in terms of the PWIA instead of the PWBA. From the Appendix (see also Refs. [9, 24]) we have for the single–nucleon longitudinal response (on–shell)

$$R_L^{[T=0,1]} \simeq (1 + \lambda)^2 \frac{G_E^{T\,2} + \tau G_M^{T\,2}}{(1 + \tau)} - \kappa^2 G_M^{T\,2} \,, \tag{45}$$

where $\lambda = \omega/2M$ and $\kappa = q/2M$, as above. For the deuteron response we have to sum over the $T = 0$ and $T = 1$ channels and integrate over momentum. Defining $\xi = \left(\frac{G_M^{T=0\,2} + G_M^{T=1\,2}}{G_E^{T=0\,2} + G_E^{T=1\,2}} \right)$, which is a large number (of order 10), we have for the $\Delta_{BSF/BN}$ ratio in the PWIA

$$\Delta_{BSF/BN} = |\frac{(1 + \lambda)^2}{(1 + \tau)}(1 + \tau\xi) - \kappa^2\xi - 1| \,. \tag{46}$$

Observe that on the quasielastic peak we have $\lambda \to \tau$ and $\kappa^2 \to \tau(1 + \tau)$ and so the G_M terms cancel and $\Delta_{BSF/BN} \to \tau$, as reflected in Fig. 11a. Away from the quasifree region, however, despite seemingly being suppressed by an extra factor of τ, these terms are important since the actual scale is not τ but $\xi\tau$. With the presence of these terms $\Delta_{BSF/BN}$ can have a ridge (it could not have one without these terms, since $(1 + \lambda)^2/(1 + \tau)$ is monotonically increasing with ω as q is kept fixed) which is approximately at

$$\lambda \simeq \frac{1}{\xi} + \kappa^2 \Rightarrow \omega \simeq \omega_{QE} + \frac{2M}{\xi} \,, \tag{47}$$

with $2m\xi \cong 100$ MeV/c, which describes rather well the ridge in Fig. 11a. The importance of the large magnetic moment in counterbalancing the small τ has been already stressed in the past [27]. We wish to draw the attention of the reader to another point. If we include the $(1 + \tau)$ factor (the Darwin term) only, the relativistic longitudinal response is *smaller* than the nonrelativistic one [3]. However, what we see from our expressions is quite different, as $(1 + \lambda)^2 \simeq \left(\frac{E_N + E_B}{2M} \right)^2$ is in fact even more important, since it is equal to $(1 + \tau)^2$ on the quasielastic peak and thus our "relativistic" longitudinal response is larger than the nonrelativistic one. In fact, by *multiplying* the nonrelativistic response, Eq. (33) by $(1 + \tau)$ and then forming a new $\Delta_{BSF/BN}$ ratio we see from Fig. 11b that we can include relativistic corrections in the quasifree region very effectively as now $\Delta_{BSF/BN}$ is of order 1% in this region.

Next we discuss relativistic corrections to the transverse response. Contours for the ratio $\Delta_{BSF/BN}$ for the transverse response are plotted in Fig. 12a. The effects are important even

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in the quasielastic region. One notices that the relativistic corrections have only a small q dependence. Let us attempt to understand this by using the PWIA formula for the transverse response

$$R_T^{[T=0,1]} \simeq \left[\frac{\kappa^2}{\tau}(1+\lambda)^2 - (1+\tau) \right] \frac{G_E^{T\,2} + \tau G_M^{T\,2}}{(1+\tau)} - 2\tau G_M^{T\,2} \ . \tag{48}$$

Here, the exchange effects are not negligible and accordingly an analysis of the $\Delta_{BSF/BN}$ ratio using the Impulse approximation in the place of the PWBA will only be a rough approximation, although this still accounts for the basic trend of the graph. Working as before we can cast the ratio $\Delta_{BSF/BN}$ in the form

$$\Delta_{BSF/BN} = \left| \frac{-\lambda^2}{\kappa} + \frac{(1+\tau\xi)}{2\kappa^2\xi(1+\tau)} \left[\frac{\kappa^2}{\tau}(1+\lambda)^2 - (1+\tau) \right] \right| \ . \tag{49}$$

The term in brackets is $(p\sin\theta/M)^2$ and vanishes on the quasielastic peak. The first term, λ^2/κ simply reflects the fact that the extreme nonrelativistic expressions in Eq. (33) have a q^2 instead of a $|Q^2|$ that appears multiplying the leading–order magnetic terms in Eq. (48). This is an example of the first class of relativistic corrections mentioned before. It results in contours parallel to the q–axis. The other term in the $\Delta_{BSF/BN}$ ratio is a second–class correction and hence is minimal on the quasielastic peak. Thus, by simply rewriting the nonrelativistic PWIA using τ instead of κ^2, the resulting $\Delta_{BSF/BN}$ ratio, plotted in Fig. 12b is now less than 1% in the quasielastic region.

6. The Quasi–Deuteron Case

From the previous sections we have seen that interference effects can be suppressed because of (a) the small Fermi momentum of the deuteron which characterizes the $\mathcal{E}/\mathcal{D}$ ratio, (b) the vanishing of G_{En} at small values of $|Q^2|$ which minimizes exchange effects in the EM longitudinal response, or (c) the factor of 1/3 in Eq. (33) which suppresses the transverse EM response. We have seen that in the case of parity–violating electron scattering (b) does not apply and the longitudinal response has a large exchange contribution. In this section we wish to investigate another situation where one can get sizable interference effects. Consider the quasi–deuteron model [33, 34, 35, 36, 37] where the scattering process can be thought as occurring off a NN quasiparticle embedded in a complex nucleus. In such a situation the spatial extent of the quasiparticle can be much smaller than a free deuteron, resulting in a broader momentum distribution, which in turn results in an $\mathcal{E}/\mathcal{D}$ ratio that tends to drop less rapidly with momentum transfer. Moreover, this quasiparticle can be in a $S = 0$, $T = 1$, $J = 0$ state, namely only an $L = 0$, S–state where the spin sums are much easier to evaluate. We obtain for the coincidence hadronic tensor:

$$D^{\mu\nu} \sim u(p_B)^2 \left\{ J_{(s)}^{*\mu} J_{(s)}^{\nu} + J_{(s)}^{*\mu} J_{(s)}^{\nu} \right\} \left[1 - \delta_{\mu,2} - \delta_{\nu,2} + 2\delta_{\mu,2}\delta_{\nu,2} \right] \ , \tag{50}$$

and similarly

$$D'^{\mu\nu} \sim u(p_N)^2 \left\{ H_{(n)}^{*\mu} H_{(n)}^{\nu} + H_{(s)}^{*\mu} H_{(s)}^{\nu} \right\} \left[1 - \delta_{\mu,2} - \delta_{\nu,2} + 2\delta_{\mu,2}\delta_{\nu,2} \right] \ . \tag{51}$$

Thus, there is no change in the direct terms (Cf. Eq. (25)). For the exchange terms one again obtains (Eq. 27) with

$$
\begin{aligned}
\mathcal{E}^{\mu\nu} \sim\ & u(p_N)u(p_B)\Big\{ J^{*\mu}_{(n)}H^{\nu}_{(n)}\Big[1-\delta_{\mu,2}-\delta_{\nu,2}\Big] \\
& -J^{*\mu}_{(s)}H^{\nu}_{(s)}\Big[1-\delta_{\mu,2}-\delta_{\nu,2}+2\delta_{\mu,2}\delta_{\nu,2}\Big]\Big\}\ .
\end{aligned}
\tag{52}
$$

We can now compute the longitudinal and transverse inclusive responses: after taking the nonrelativistic limit we obtain the analog of Eq. (33).

$$
\begin{aligned}
R_L \sim\ & \left\{ G_E^{T=0\,2}\big(\mathcal{D}+\mathcal{E}\big) + G_E^{T=1\,2}\big(\mathcal{D}-\mathcal{E}\big)\right\} \\
R_T \sim\ & \kappa^2\left\{ G_M^{T=0\,2}\big(\mathcal{D}-\mathcal{E}\big) + G_M^{T=1\,2}\big(\mathcal{D}+\mathcal{E}\big)\right\}\ .
\end{aligned}
\tag{53}
$$

That is, the extreme nonrelativistic limit of the longitudinal response remains functionally the same (the $\mathcal{D}$ and $\mathcal{E}$ integrals change due to the different momentum distribution), but the transverse response exchange terms are different and there is no suppression factor of $1/3$ as in Eq. (33) — this is due to the fact that for a spin–0 target the same number of diagrams contributes to both the direct and the exchange terms. These points are made clear in Figs. 13– 15 where we show the analogs of Figs. 8–10 using Eq. (52) and a quasi–deuteron square–well momentum distribution with Fermi momentum 300 MeV/c and well–depth 50 MeV. The $\mathcal{E}/\mathcal{D}$ ratio remains significant up to rather high momentum transfer values in the quasifree region, dropping to less than 20% only after 500 MeV/c, whereas for the usual $^3S_1$ deuteron discussed above the ratio drops below 20% already at about 200 MeV (inside the quasielastic ridge). The longitudinal response does not show any significant change since the neutron form factor suppressing effect is still present. However, the transverse response (Fig. 15) shows a dramatic change compared to (Fig. 10) corresponding to the deuteron, reflecting in a linear fashion the $\mathcal{E}/\mathcal{D}$ ratio (this is because $G_M^{T=0\,2} \ll G_M^{T=1\,2}$).

7. Summary and Conclusions

In this work we have relaxed one of the assumptions inherent in the plane–wave impulse approximation to electron–nucleus scattering, namely, the identification of the detected nucleon with the nucleon struck by the virtual photon, while retaining specific aspects of relativity. This we call the relativistic plane–wave Born approximation. For inclusive electron scattering from deuterium (where we integrate over the detected nucleon's quantum numbers) the difference between the PWBA and the PWIA amounts to the interference between the amplitudes corresponding to the case where the struck and detected nucleons are the same and the one where they are not. We calculate the hadronic tensor in the PWBA treating the single–nucleon matrix elements relativistically and the deuteron wave function nonrelativistically. The relativistic effects are governed by two dimensionless scales, the first set by the ratio $\kappa = q/2M$ which characterizes the single–nucleon matrix elements and the second by the ratio $\eta_F = p_F/M$ which enters whenever the ground–state deuteron wave function occurs as in the matrix elements making up the deuteron break–up amplitude. Our objective here has been to explore the degree to which quasifree electrodisintegration at high q and ω decouples into **relativistic** effects

where expansions in terms of κ cannot be undertaken and **exchange** effects whose importance is governed by η_F/κ.

As far as the interference (exchange) terms are concerned, our results can be summarized as follows: These terms are proportional to the momentum–space integral of the overlap between deuteron wave functions centered at momenta corresponding to the kinematics for the two outgoing nucleons. Since these momenta differ significantly for large momentum transfer values, the overlap is small and consequently the interference effects are also small. Moreover, the leading nonrelativistic contribution to the interference effects is proportional to $G_E^p G_E^n$ for the longitudinal response and $\frac{1}{3}G_M^p G_M^n$ for the transverse response. Thus, when the overlap is significant (low q), the electric neutron form factor is negligible, and the longitudinal contribution to the interference terms is very small. On the other hand, the transverse contributions are suppressed as well, however now because of an extra factor of $1/3$ of purely geometrical (*i.e.,* Clebsch-Gordan) origin. Therefore, at least insofar as the nonrelativistic analysis is concerned, it appears that the interference contributions to inclusive deuteron electrodisintegration are usually relatively mild, except for backward–angle scattering and near threshold. These simple nonrelativistic arguments appear also to be borne out in the present relativistic PWBA framework.

As far as the relativistic effects themselves are concerned, our analysis which retains terms of all orders in κ yields sizeable differences when compared with a nonrelativistic expansion, as expected, especially when kinematics beyond the quasifree region are explored. This is true in the relativistic PWIA and continues to be true in the relativistic PWBA.

Thus, we see that indeed the interference and relativistic effects largely decouple, especially in the longitudinal response, the former usually being important only when the momentum transfer is small, that is, for $\kappa < \text{few}\times\eta_F$ corresponding to $q < \text{few}\times(2p_F)$. Since the latter become important when $\kappa \sim 1$, the effects usually decouple as long as $p_F \ll M$ ($\eta_F \ll 1$), which is certainly true for the deuteron where $p_F \cong 55 \text{ MeV/c}$ (in contrast to heavier nuclei where it is larger — see below). There are exceptions to these statements, however, and we have identified two cases where the interference effects are important even at high momentum transfers. The first is parity–violating electron scattering where, due to the interesting coincidence that $1 - 4\sin^2\theta_W \approx 0$, the interference effects for the longitudinal PV response are enhanced by an order–of–magnitude. A second is the quasi–deuteron model, where the wave function overlap is significant even at high q due to the larger Fermi momentum for nuclei in general compared with deuterium which is exceptional (for example, $\eta_F(^{40}\text{Ca})/\eta_F(^2\text{H}) \cong 4.5$). Effectively then the conditions $\kappa \sim \text{few}\times\eta_F$ and $\kappa \sim 1$ can be met simultaneously. The largeness of the overlap also stems in part from the spin–0 nature of the quasi–deuteron which eliminates the factor of $1/3$ in R_T occurring in the spin–1 deuteron case, thus leading to an important transverse interference contribution. These observations derived from our exploratory study of the relativistic PWBA within the context of a simple quasi–deuteron model suggest further investigations of nuclei with $A > 2$. It is our intent to return in future work to study both exceptions in more detail.

Appendix

Let us first discuss Eq. (23). We introduce the dimensionless variables $\boldsymbol{\kappa} = \boldsymbol{q}/2M$, $\boldsymbol{\eta} = \boldsymbol{p}/M$, $\bar{\lambda} = \bar{\omega}/2M$, $\varepsilon = \sqrt{\boldsymbol{p}^2 + M^2}/M$ and $\tau = (q^2 - \bar{\omega}^2)/4M^2$, where we define $\bar{\omega} = E_N - E_B = \omega + M_d - 2M\varepsilon$ (the energy transferred to an on–shell nucleon with momentum $\mathbf{p}$). We also introduce two angles Ψ and Φ:

$$
\begin{aligned}
\tan\Psi &\equiv \frac{|\boldsymbol{\kappa} \times \boldsymbol{\eta}|}{1 + \tau + \varepsilon + \bar{\lambda}} \\
\tanh\Phi &\equiv \frac{|\boldsymbol{\kappa} \times \boldsymbol{\eta}|}{\sqrt{\tau}(\varepsilon + \bar{\lambda})} \; .
\end{aligned}
\tag{54}
$$

Then for the charge operator we obtain from Ref. [27]

$$
\begin{aligned}
J_n^{(0)} &= \frac{1}{\sqrt{\varepsilon(\varepsilon + 2\bar{\lambda})]}} \frac{\kappa}{\sqrt{\tau}} \Big[G_E^{\tau_N} \cosh\Phi \cos\Psi + \sqrt{\tau} G_M^{\tau_N} \sinh\Phi \sin\Psi \Big] \\
J_s^{(0)} &= \frac{-1}{\sqrt{\varepsilon(\varepsilon + 2\bar{\lambda})}} \frac{\kappa}{\sqrt{\tau}} \Big[-G_E^{\tau_N} \cosh\Phi \sin\Psi + \sqrt{\tau} G_M^{\tau_N} \sinh\Phi \cos\Psi \Big] \; .
\end{aligned}
\tag{55}
$$

For the transverse projections we have

$$
\begin{aligned}
J_n^{(1)} &= \frac{1}{\sqrt{\varepsilon(\varepsilon + 2\bar{\lambda})}} \Big[G_E^{\tau_N} \sinh\Phi \cos\Psi + \sqrt{\tau} G_M^{\tau_N} \cosh\Phi \sin\Psi \Big] \\
J_s^{(1)} &= \frac{-1}{\sqrt{\varepsilon(\varepsilon + 2\bar{\lambda})}} \Big[-G_E^{\tau_N} \sinh\Phi \sin\Psi + \sqrt{\tau} G_M^{\tau_N} \cosh\Phi \cos\Psi \Big] \\
J_n^{(2)} &= \frac{-i}{\sqrt{\varepsilon(\varepsilon + 2\bar{\lambda})}} G_M^{\tau_N} \sqrt{(\sqrt{\tau} \cosh\Phi \sin\Psi + \sinh\Phi \cos\Psi)^2 - (1 + \tau) \sinh^2\Phi} \\
J_s^{(2)} &= \frac{-i}{\sqrt{\varepsilon(\varepsilon + 2\bar{\lambda})}} G_M^{\tau_N} \Big[\sqrt{\tau} \cosh\Phi \cos\Psi - \sinh\Phi \sin\Psi \Big] \; .
\end{aligned}
\tag{56}
$$

The longitudinal ($\mu = 3$) projection of the 3–current is related to the charge by the continuity equation

$$
\kappa J_i^{(3)} = \boldsymbol{\kappa} \cdot \boldsymbol{J}_i = \lambda J_i^{(0)} \; .
\tag{57}
$$

The H–type matrix elements in Eq. (23) are obtained by replacing $\boldsymbol{p}_B$ with $-\boldsymbol{p}_N$, τ_N with τ_B and λ with $-\lambda$. It is straightforward to check that this results in just reversing the sign of $\sin\Psi$ and $\sinh\Phi$. The extreme nonrelativistic limit (that is, to $\mathcal{O}[1/M]^0$) of Eqs. (54–56) is presented in Table II, whereas a higher–order nonrelativistic reduction can be found in Ref. [27].

We now turn to Eq. (28). The coefficients $a_{i,j}^{K,X}$ are defined as follows:

$$
\begin{aligned}
a_{i,j}^{K,X} = \sum_{M_S, M_S'} \sum_{m,m'} \sum_{m_N, m_B} (-1)^{1+M_S} \begin{pmatrix} 1 & 1 & K \\ M_S & -M_S' & X \end{pmatrix} \times \\
\begin{pmatrix} \frac{1}{2} & 1/2 & 1 \\ m_N & m' & -M_S' \end{pmatrix} \begin{pmatrix} 1/2 & 1/2 & 1 \\ m_B & m & -M_S \end{pmatrix} \theta_{i,j} \, g_{m'}^{\mu} \, g_m^{\nu} \; ,
\end{aligned}
\tag{58}
$$

20

where we have set

$$
\begin{aligned}
\theta_{n,n} &\equiv \delta_{m_B,m'}\delta_{m,m_N} & (59)\\
\theta_{n,s} &\equiv 2m\delta_{m',m_B}(1-\delta_{m,m_N})\\
\theta_{s,n} &\equiv 2m'\delta_{m,m_N}(1-\delta_{m',m_B})\\
\theta_{s,s} &\equiv 4mm'(1-\delta_{m,m_N}-\delta_{m',m_B}+\delta_{m',m_B}\delta_{m,m_N}) \ ,
\end{aligned}
$$

with

$$
g_m^\mu = \begin{cases} 1 & , \text{ if } \mu \neq 2 \\ 2m & , \text{ if } \mu = 2. \end{cases} \tag{60}
$$

No summation convention over repeated indices is implied here. In Table I we write down explicitly the coefficients $a_{i,j}^{K;X}$. We use a block–diagonal matrix notation in order to bring out the different behaviour of the (2)–components as seen in Eq. (24).

Finally, let us calculate the longitudinal and transverse responses in the S–state–only limit using Eqs. (25–28) and Table I. For the longitudinal response we obtain

$$
\begin{aligned}
R_L^{S-state} \sim \ & \left[|J_{(n)}^0|^2 + |J_{(s)}^0|^2\right]|u(p)|^2 + \left[|H_{(n)}^0|^2 + |H_{(s)}^0|^2\right]|u(p_N)|^2 \\
& +2u(p)u(p_N)\Re\left[\{H_{(n)}^{*0}J_{(n)}^0\} + \left(\frac{1}{3}\right)\{H_{(s)}^{*0}J_{(s)}^0\}\right] \ . \tag{61}
\end{aligned}
$$

Similarly, for the transverse response we have

$$
\begin{aligned}
R_T^{S-state} \sim \ & \left[|J_{(n)}^1|^2 + |J_{(s)}^1|^2 + |J_{(n)}^2|^2 + |J_{(s)}^2|^2\right]|u(p)|^2 \\
& +\left[|H_{(n)}^1|^2 + |H_{(s)}^1|^2 + |H_{(n)}^2|^2 + |H_{(s)}^2|^2\right]|u(p_N)|^2 \\
& +2u(p)u(p_N)\Re\left[\{H_{(n)}^{*1}J_{(n)}^1\} + \left(\frac{1}{3}\right)\{H_{(n)}^{*2}J_{(n)}^2\}\right. \\
& \left. + \left(\frac{1}{3}\right)\{H_{(s)}^{*1}J_{(s)}^1\} + \left(\frac{1}{3}\right)\{H_{(s)}^{*2}J_{(s)}^2\}\right] \ . \tag{62}
\end{aligned}
$$

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Table I The $a_{i,j}^{K,X}$ coefficients $\begin{pmatrix} \mu,\nu \neq 2 & \nu = 2 \\ \mu = 2 & \mu = \nu = 2 \end{pmatrix}$				
(K,X)	NN	NS	SN	SS
$(0,0)$	$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$	0	0	$\frac{1}{3\sqrt{3}}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
$(1,-1)$	0	$\frac{1}{3\sqrt{3}}\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$	$\frac{1}{3\sqrt{3}}\begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix}$	0
$(1,0)$	$\frac{\sqrt{2}}{3\sqrt{3}}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	0	0	0
$(1,+1)$	0	$\frac{1}{3\sqrt{3}}\begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$	$\frac{1}{3\sqrt{3}}\begin{pmatrix} -1 & 0 \\ -1 & 0 \end{pmatrix}$	0
$(2,-2)$	0	0	0	$\frac{1}{3\sqrt{5}}\begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix}$
$(2,-1)$	0	$\frac{1}{3\sqrt{5}}\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$	$\frac{1}{3\sqrt{5}}\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}$	0
$(2,0)$	$\frac{2\sqrt{2}}{3\sqrt{15}}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	0	0	$-\frac{\sqrt{2}}{3\sqrt{15}}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
$(2,+1)$	0	$\frac{1}{3\sqrt{5}}\begin{pmatrix} 0 & 0 \\ 1 & -1 \end{pmatrix}$	$\frac{1}{3\sqrt{5}}\begin{pmatrix} 0 & -1 \\ 0 & -1 \end{pmatrix}$	0
$(2,+2)$	0	0	0	$\frac{1}{3\sqrt{5}}\begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix}$

Table II Extreme Nonrelativistic Limit of Eq. (23)				
(μ)	$J_{(n)}^{\mu}$	$H_{(n)}^{\mu}$	$J_{(s)}^{\mu}$	$H_{(s)}^{\mu}$
(0)	$G_E^{\tau_N}$	$G_E^{\tau_B}$	0	0
(1)	0	0	$-\kappa\, G_M^{\tau_N}$	$-\kappa\, G_M^{\tau_B}$
(2)	0	0	$i\kappa\, G_M^{\tau_N}$	$i\kappa\, G_M^{\tau_B}$

Table III.1	The $\mathcal{D}$ and $\mathcal{E}$ integrals at $q = 150$ MeV/c		
ω (MeV)	$\mathcal{D}$	$\mathcal{E}$	$\mathcal{E}/\mathcal{D}$
8.3	0.939	0.912	0.971
10	5.076	3.064	0.604
16	8.930	2.563	0.287
26	2.943	0.975	0.331
36	1.107	0.429	0.388

Table III.2	The $\mathcal{D}$ and $\mathcal{E}$ integrals at $q = 300$ MeV/c		
ω (MeV)	$\mathcal{D}$	$\mathcal{E}$	$\mathcal{E}/\mathcal{D}$
27	0.100	0.079	0.784
30	0.285	0.136	0.475
40	2.835	0.301	0.106
50	4.846	0.208	0.043
100	0.222	0.003	0.014

Figure Captions

1. Kinematics for single–arm coincidence electron scattering. Besides the outgoing electron K^μ, the nucleon labelled P_N^μ is also detected.

2. Electrodisintegration of deuterium in the PWBA, with nucleon N detected in coincidence. (a): Amplitude corresponding to the PWIA where the detected nucleon (N) is the one that reacted with the virtual photon. (b): While the detected nucleon is still nucleon (N), it is nevertheless nucleon B that has reacted. This amplitude is ignored in the PWIA.

3. The hadronic tensor for inclusive deuteron electrodisintegration in the PWBA. The momenta and spins of the nucleons involved are indicated.

4. The Direct ($\mathcal{D}$) and exchange ($\mathcal{E}$) integrals at $q = 150$ MeV/c: (a) near threshold ($\omega = 9$ MeV) and (b) on the quasielastic peak ($\omega = 14.13$ MeV).

5. The ratio $\mathcal{E}/\mathcal{D}$ as a function of momentum transfer ω for (a) $q = 150$ MeV/c and (b) $q = 300$ MeV/c. The arrows mark the position of the quasielastic peak in each case.

6. The $\mathcal{E}/\mathcal{D}$ ratio as a function of the momentum transfer q for energy transfer fixed to the quasielastic peak (Eq. (40)).

7. Forward angle ($\theta_e = 35^o$) parity–violating asymmetry in the PWBA (solid lines) and PWIA (dashed lines) as a function of the energy transfer ω for fixed momentum transfer: (a) $q = 150$ MeV/c and (b) $q = 300$ MeV/c. The position of the quasielastic peak is denoted by an arrow in each case.

8. Percentage contours for the absolute value of the ratio $\mathcal{E}/\mathcal{D}$ in the $q-\omega$ plane. The shaded area corresponds to quasifree kinematics calculated according to the Fermi–Gas formula with $p_F = 60$ MeV/c (see text).

9. Percentage contours for the difference between PWIA and PWBA as represented by the ratio $\Delta_{BF/IF}$ (Eq. (43)) for the longitudinal response R_L in inclusive electrodisintegration of deuterium.

10. Same as in Fig. 9, except now for the transverse response R_T.

11a. Contour plot for the ratio $\Delta_{BSF/BN}$ (Eq. (44)) in the case of the longitudinal response.

11b. Same as in Fig. 11a, but with the PWBA modified by $(1 + \tau)$ (see text).

12a. Same as in Fig. 11a, except now for the transverse response R_T.

12b. Same as in Fig. 12a, but with the PWBA modified by τ/κ^2 (see text).

13. Same as in Fig. 8, but with artificial quasi–deuteron $J = 0$ wave functions corresponding to $p_F = 300$ MeV/c and binding energy $E_b = 16$ MeV.

14. Same as in Fig. 9, except now for the quasi–deuteron.

15. Same as in Fig. 10, except now for the quasi–deuteron.

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